response curve since with a nonlinear curve the value given will be neither an average nor the value at a particular yaw angle. It would therefore be advantageous to redefine the yaw meter sensitivity C_ψ which would be appropriate when the yaw meter is kept stationary and the flow angle is evaluated by using a calibration curve. In this case it would be useful to know the average sensitivity over the range of yaw angles being considered, viz.

$$C_\psi = \int_{\psi_1}^{\psi_2} \frac{C_p d\psi}{(\psi_2 - \psi_1)} \tag{5}$$

It can be seen that, in the case where the yaw meter response is linear and the lower limit of yaw (ψ_1) is zero, the expression given in Eq. (5) reduces to that of Eq. (4) as should be expected.

Conclusions

A definition of yaw meter sensitivity has been given which is appropriate to the commonly used "null reading" technique [Eq. (4)], and it is proposed that the symbol C_{ψ_0} should be adopted for this definition.

In addition, a more precise definition of the yaw meter sensitivity C_ψ has been formulated for general comparison between different types of yaw meter where the instrument is held stationary and the flow angle is determined from a calibration curve.

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Effect of Hydrostatic Pressure on Underwater Towed Body Cable Configurations

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IN Ref. 1, Goodman and Breslin show that the effect of hydrostatic pressure on the static equilibrium equations of a cable in a stream can be completely accounted for by replacing the tension T in the classical equations by the effective tension Te, where

$$Te = T + \frac{\rho g A_0 z}{(1+\epsilon)} \tag{1}$$

and by multiplying the stream-induced normal and tangential drag forces by the factor $(1 + \frac{1}{2}\epsilon)$.

Here ρ is the mass density of the fluid, g is the acceleration due to gravity, A_0 is the cross-sectional area of the unstretched cable, z is the depth of the cable element below the free surface, ϵ is the extension per unit length of the cable (strain) defined by

$$\epsilon = \frac{1}{E}\left[\frac{T(1+\epsilon)}{A_0} + \rho g z\right] \tag{2}$$

and E is Young's modulus.

Generally, the tension and cable angle φ are known at one end of the cable, and, since the dependent variables of the

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Table 1 Static solution from first iteration which assumes that the actual tension = effective tension

Relaxed cable length, ft	Stretched cable length, ft	Depth below surface, ft	Excursion from ship, ft	Effective tension, lb	Actual tension, lb	Cable angle, deg	Cable strain	
0.00	0.00	63.49	107.50	247.44	247.44	69.96	.00079	(Body)
9.85	9.86	54.58	103.32	243.71	243.71	60.03	.00078	
19.70	19.71	46.45	97.76	240.76	240.76	51.38	.00077	
29.55	29.57	39.18	91.12	238.52	238.52	44.07	.00076	
39.39	39.42	32.72	83.68	236.89	236.89	37.96	.00076	
49.24	49.28	27.02	75.64	235.78	235.78	32.86	.00075	
59.09	59.14	22.00	67.16	235.11	235.11	28.56	.00075	
68.94	68.99	17.59	58.36	234.82	234.82	24.91	.00075	
78.79	78.85	13.68	49.31	234.86	234.86	21.76	.00075	
88.64	88.70	10.25	40.07	235.18	235.18	19.01	.00075	
98.48	98.56	7.24	30.69	235.76	235.76	16.58	.00075	
108.33	108.42	4.61	21.19	236.57	236.57	14.40	.00075	
118.18	118.27	2.33	11.60	237.59	237.59	12.43	.00076	
128.03	128.13	0.36	1.94	238.79	238.79	10.61	.00076	
130.00	130.10	0.00	0.00	239.06	239.06	10.27	.00076	(Ship)

Table 2 Results from final iteration—converged static solution

Relaxed cable length, ft	Stretched cable length, ft	Depth below surface, ft	Excursion from ship, ft	Effective tension, lb	Actual tension, lb	Cable angle, deg	Cable strain	
0.00	0.00	64.39	107.04	253.05	247.44	69.96	.00081	(Body)
9.85	9.86	55.46	102.88	249.31	244.48	60.24	.00079	
19.70	19.71	47.31	97.35	246.34	242.22	51.74	.00079	
29.55	29.57	39.99	90.77	244.07	240.58	44.52	.00078	
39.39	39.42	33.47	83.38	242.39	239.47	38.46	.00077	
49.24	49.28	27.70	75.39	241.23	238.81	33.38	.00077	
59.09	59.14	22.60	66.96	240.51	238.54	29.09	.00077	
68.94	68.99	18.09	58.20	240.17	238.59	25.44	.00077	
78.79	78.85	14.11	49.18	240.15	238.92	22.29	.00077	
88.64	88.71	10.60	39.97	240.43	239.50	19.53	.00077	
98.48	98.56	7.51	30.62	240.96	240.30	17.09	.00077	
108.33	108.42	4.79	21.14	241.72	241.30	14.91	.00077	
118.18	118.27	2.43	11.58	242.68	242.47	12.93	.00077	
128.03	128.13	0.37	1.94	243.84	243.80	11.12	.00078	
130.00	130.10	0.00	0.00	244.09	244.09	10.78	.00078	(Ship)

classical equilibrium equations are the tension and cable angle, this information is sufficient to integrate the cable equations as an initial value problem and hence to determine the complete configuration. However, when the effect of hydrostatic pressure is taken into consideration, the dependent variables become the effective tension and the cable angle. Since it is the actual tension and cable angle which are known at one end of the cable, this information is generally not sufficient to be able to integrate the cable equations unless the depth at the end where the data are supplied is also known so that the initial value of the effective tension can be determined.

In the case of a towed underwater body, the tension and cable angle are known at the body, while the depth of the other end of the cable, which is attached to the towing ship, is known. The initial value problem of the classical equations has, therefore, been transformed into a two-point boundary value problem and the depth of a cable element z now becomes a third dependent variable. The equation for the depth.

$$\frac{dz}{ds} = (1+\epsilon)\sin\varphi \tag{3}$$

has become coupled with the equations for the effective tension and cable angle, and all three equations, together with their boundary conditions, must be satisfied simultaneously.

The scheme used to solve this two-point boundary value problem is an iterative one. In the first iteration we assume the value of the effective tension at the towed body to be equal to the given value of the tension and integrate to the other end of the cable where the value of z is specified. Then, by monitoring z along the cable we can determine a first approximation to the depth of the towed body cable attachment point. From this we calculate a second approximation to the effective tension at the towed body using Eq. (1). The computation up the cable is then repeated iteratively until convergence. The first iteration is, of course, the result that would be obtained if the effect of hydrostatic pressure were ignored altogether while the last iteration fully accounts for the effect of hydrostatic pressure. A sample calculation in water is given in Tables 1 and 2 using a Runge-Kutta-Gill integration scheme. The calculation is based on the input data in Table 3.

It can be seen that the effect of hydrostatic pressure is to place the towed body slightly below and forward of the position predicted by the classical theory; furthermore, the

Table 3 Input data for tow configuration

Tension at body, lb	247.44
Cable angle at body, deg	69.96
Tow speed, fps	10.00
Length of cable, ft	130.0
Depth of cable at tow ship, ft	0.0
Cable diameter, in.	1.0
Cable material	steel

tension in the cable at the tow ship attachment point is slightly increased. The latter effect indicates that ignoring hydrostatic pressure may lead to a nonconservative design.

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Similarity in the Modeling of Cable Twisting and Looping

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I. Introduction

ORDINARY armored cable generates a twist moment under tension. An assembly of armored and armorless cables therefore represents a twist moment and twist deformation problem as well as a tension problem. Under certain conditions involving moment and tension, a section of cable may relieve twist by looping. If the yield stress of the armor wires is exceeded by subsequent pulling of a looped section, a kink will result.

In the installation of communications cables in deep water, the twist properties of the coaxial cable are important.[1] Excessive twist deformation can produce cable failure. Kinks in cable make recovery difficult or impossible.

Model tests which duplicate the full-scale installation appear attractive for making comparisons of different laying schemes and determining the conditions under which looping and fouling are likely. In order for model tests to be valid, certain conditions must be satisfied by the model. These conditions are embodied in the dimensionless similarity parameters, which must have the same value in the model as in the full-scale operation.

In Ref. 2, the similarity parameters were worked out for cases in which twist, bending, looping, and kinking are not important. More precisely, the shear forces associated with twist and bending are small compared to the tension. Only cable weight and tangential and normal drag appear in the equilibrium equations. This is the case for most cable configuration problems.

The additional considerations lead to more terms in the equations defining the configurations, and hence to additional similarity parameters. The general problem is impossible to solve analytically, except for highly simplified cases[3] and impractical to model. As pointed out in Ref. 2, however, twist and bending ordinarily have a strictly localized effect on the cable tension and configuration. This suggests that the general problem is divided into two sequential problems. First, the cable tension and configuration would be worked out, either analytically, or by models using the parameters of Ref. 2. Then twist and bending effects would be superposed, modeling only the regions in which these effects are important.

II. The Equations of Equilibrium

The equations of equilibrium for a segment of cable ds, as shown in Fig. 1, include three force equations and three moment equations. These constitute the starting point of the "thin-rod" theory of elasticity.[4]

One possible way to specify the deformation of a bent and twisted rod is to make use of a moving coordinate system. First it is assumed that the rod is laid in an unbent and untwisted state. The z' axis is made coincident with the central axis of the rod. The x' and y' axes form a right-handed system and are aligned with the principal axes of the cross section of the rod. If the cross section has axial symmetry, as in most cables, the attitude of the x' axis is arbitrary.

The moving coordinate system x, y, z, is defined for each location s in the deformed rod as follows. The moving coordinate system translates and rotates in such a way that, at any time, the z axis is parallel to the cable axis, and the x and y coincide with what were the x' and y' axes in the undeformed rod. The angular velocity of the x, y, z, frame when the rod is traversed at unit speed has the components κ, κ' and τ along the x, y and z axes. Thus κ and κ' are bending rates per unit length and τ is the twist rate per unit length.

Let the components of force along the x,y,z axes be N,N' and T; N and N' are shear forces and T is the tension. Let the components of the moment vector be G,G' and H; G and G' are bending moments and H is the twist moment. Assume that the curvature and the twist can be related to the moments through the bending resistances A and B, and the twist resistance C, as follows:

$$G=A\kappa,\ G'=B\kappa',\ H=C\tau. \tag{1}$$

If the cross-section is axially symmetric, A and B are identical.

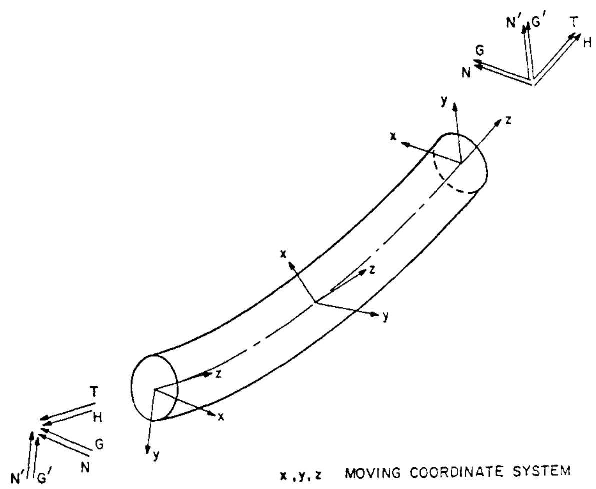

Fig. 1 Forces and moments on a segment of cable.

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